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On the implementation of effects of Lorentz force in turbulence closure models

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Abstract

The effects of Lorentz force on turbulent flow of conductive fluid are analysed within the framework of second-moment and eddy-viscosity closure models. Additional terms representing the magnetohydrodynamic interactions in the transport equations for the turbulent stress tensor and energy dissipation rate are derived in the exact form and the parts of the terms that cannot be treated exactly are then modelled. The modelling is based on the term-by-term analysis of the direct numerical simulations (DNS) of turbulent flow in an infinite plane channel subjected to transverse uniform magnetic field (Noguchi, H., Ohtsubo, Y., Kasagi, N., 1998. DNS database of turbulence and heat transfer). A priori validation of the new model is presented and compared with the DNS results for the Reynolds number based on the friction velocity $Re_\tau = 150$ and for the Hartmann number $Ha = 6$. The same approach is then followed to modify the the low-Re number k–ε model. The new k–ε model has been applied to solve the developing three-dimensional flow of mercury in a rectangular-sectioned duct at inlet Reynolds number $Re = 2 \times 10^5$, subjected over a part of its length to a magnetic field with $Ha = 700$ (corresponding to a Stuart number $N = 2.45$). The predicted development of 'M' shaped velocity profiles shows acceptable agreement with the experiments of Tananaev (1978) and visible improvement in comparison with the eddy-viscosity model of Ji and Gardner (Ji, H.C., Gardner, R.A., 1997. International Journal of Heat Mass Transfer 40, 1839–1851).

Keywords: Magnetofluid dynamics; Turbulence; Closure models

Notation

$Re = UL/\nu$	mean flow Reynolds number
$Re_\tau = U_\tau L/\nu$	mean flow Reynolds number based on friction velocity
$Re_m = \mu_0 \sigma UL$	magnetic Reynolds number
$Ha = B_0 L\sqrt{\sigma/\rho\nu} = \sqrt{NRe}$	Hartmann number
$N = \sigma B_0^2 L/\rho U$	Stuart or interactive number
$\mathbf{U}$	velocity vector (m/s)
$\mathbf{E}$	electric field intensity (V/m)
$\mathbf{J}$	electric current density (A/m^2)
Φ	electric potential (V)
$\mathbf{B}$	magnetic field induction (T)
L	characteristic dimension (m)
k	turbulence kinetic energy (m^2/s^2)
$\overline{u_i u_j}$	Reynolds stress components (m^2/s^2)
C_i^{M}, C_ϕ	constants in turbulence model
S_i^{M}	magnetic source terms contribution
f_μ, f^{M}	dumping functions in turbulence model

Greeks

ρ	incompressible fluid density (kg/m^3)
ν	kinematic viscosity (m^2/s)
ν_t	turbulent viscosity
σ	electric conductivity (1/Ω m)
μ_0	magnetic permeability (Ω s/m)
ε	dissipation rate of turbulence kinetic energy (m^2/s^3)
ϵ_{ijk}	unit tensor of third order

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1. Introduction

The application of a magnetic field in controlling flow, heat and mass transfer is regarded nowadays as an important technological advance towards the improvement of quality and increase in productivity in various metallurgical processes. In the casting of metals, a stationary magnetic field is used as a brake to suppress instabilities and to control the flow conditions in tundish devices, or a nonstationary field is applied to enhance mixing during casting. Another prospective application is in semiconductor crystal growth, where the control and suppression of turbulence is an important prerequisite for achieving the desired quality, particularly in view of the new trend to use larger crucibles. In metal casting applications the Lorentz force generated by the interaction of fluid velocity and applied magnetic field is the major external force. However, in crystal growth the Lorentz force also interacts with the thermal and mass buoyancy, as well as with forces generated by the crucible and/or crystal rotation.

Predicting the flow of melt, and the heat and mass transfer within it (and at its boundaries) under the influence of magnetic forces, is an important prerequisite for the design of an optimum magnetic field and for the prediction of the product quality, Branover and Lykodudis (1983). The flow is usually turbulent (metal casting) or transitional (crystal growth), so that the prediction model needs to account accurately for the effect of magnetic force on turbulence suppression or enhancement. In most full-scale applications, the melted metal assumes a complex three-dimensional domain and the process is further complicated by solidification and associated interfacial phenomena, which require accurate prediction of the phenomena near a solidifying interface. The MHD turbulent flows are characterised by the flow reorganisation caused by alignment of the vortex structures with the direction of the imposed magnetic field, which increase the anisotropy of Reynolds stresses. These structural adjustments cannot be reproduced by the Reynolds-averaged approach but its effects on damping the turbulence can be modelled using the available turbulence parameters.

This paper deals with the modelling of turbulent flows of conducting fluid under the effects of magnetic field. The goal is to introduce and validate adequate modifications for the effects of Lorentz force into engineering-type turbulence models (two-equation eddy-viscosity and algebraic stress/flux models), which would be suitable for application to complex metallurgical problems. Instead of ad-hoc modifications, the 'engineering' model is based on the truncation of a general second-moment closure, extended to account for the Lorentz force through a unified approach in modelling the effects of external forces. The model is based on a term-by-term analysis of DNS results for a fully-developed plane channel flow of liquid metal under a transverse magnetic field (Noguchi and Kasagi, 1994; Noguchi et al., 1998). In addition to this generic flow case, the paper presents also some results of modelling of a more complex three-dimensional case such as flow of conductive fluid in a square-sectioned duct subjected to local finite-length transverse magnetic field.

2. Governing equations and effects of Lorentz force

The system of equations describing the motion of electrically conducting fluid in a magnetic field consists of the Navier–Stokes momentum equations, Maxwell's equations and Ohm's law for moving media. The coupling of the equations is through the Lorentz force in the momentum equation defined as $\mathbf{F}^{\mathrm{L}} = \mathbf{J} \times \mathbf{B}$, where $\mathbf{J}$ is the total electric current density and $\mathbf{B}$ is the imposed magnetic flux density ("magnetic field"). By applying the Ohm's law for moving media, $\mathbf{J} = \sigma(-\nabla\Phi + \mathbf{U} \times \mathbf{B})$, the Lorentz force can be written in the following form, using index notations:

$$F_i^{\mathrm{L}} = \sigma\left(-\epsilon_{ijk} B_k \frac{\partial \Phi}{\partial x_j} + U_k B_i B_k - U_i B_k^2 \right), \tag{1}$$

where Φ is the electric potential and ϵ_{ijk} is the permutation symbol. Using Kirchhoff continuity condition $\nabla \cdot \mathbf{J} = 0$ leads to the numerically convenient Poisson equation for the electric potential

$$\nabla^2 \Phi = \nabla \cdot \mathbf{U} \times \mathbf{B}. \tag{2}$$

It is important to note that in these derivations the inductionless approximation was applied, i.e., effects of the fluid flow on the magnetic field were neglected. This will be the case when the magnetic Reynolds number is very small, $Re_m \ll 1$, i.e., when the electromagnetic fluctuations are driven only by the hydrodynamic turbulence (Branover, 1978; Moreau, 1990). In this way, instead of three equations for the components of magnetic field, the system of Maxwell's equation was reduced to one scalar equation for the electric potential.

Hence, for Reynolds averaged motion the momentum equation can be written as

$$\frac{\mathrm{D}U_i}{\mathrm{D}t} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\nu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \overline{u_i u_j} \right] + F_i^{\mathrm{L}}, \tag{3}$$

where the second velocity moment $\overline{u_i u_j}$ is the turbulent stress tensor.

In addition to direct interaction with the mean velocity field through the electromagnetic force (F_i^{L}), which will itself influence the turbulence through the deformed mean rate of strain, the magnetic field affects also the velocity fluctuations by the fluctuating Lorentz force $\mathbf{f}^{\mathrm{L}}$, even if the magnetic flux density $\mathbf{B}$ is assumed invariable with time. In this case the fluctuating Lorentz force reduces to (terms including the fluctuations of the magnetic field $\mathbf{b}$ are omitted)

$$f_i^{\mathrm{L}} = \sigma\left(-\epsilon_{ijk} B_k \frac{\partial \phi}{\partial x_j} + u_k B_i B_k - u_i B_k^2 \right). \tag{4}$$

We begin the analysis by considering the Reynolds-averaged transport equation for the turbulent stress tensor, which can be written in a symbolic form

$$\frac{\mathrm{D}\overline{u_i u_j}}{\mathrm{D}t} = P_{ij} + D_{ij}^{\mathrm{p}} + D_{ij}^{\mathrm{t}} + D_{ij}^{\nu} + \Phi_{ij} + S_{ij}^{\mathrm{M}} - \varepsilon_{ij}, \tag{5}$$

where P_{ij} is the stress production by the mean rate of strain, $D_{ij} = D_{ij}^{\nu} + D_{ij}^{\mathrm{t}} + D_{ij}^{\mathrm{p}}$ is the diffusion (viscous + turbulent by velocity and pressure fluctuations), Φ_{ij} is the stress redistribution by pressure scrambling effect, ε_{ij} is the stress dissipation rate, and S_{ij}^{M} is the net production of the stress tensor due to magnetohydrodynamic interaction.

The exact expression for each term in Eq. (5) for forced turbulent flows without body forces can be found elsewhere. It suffices here to recall that within the second-moment closure framework, P_{ij} can be treated in exact form, whereas the other terms need to be modelled. Various forms and levels of modelling of D_{ij}^{t}, D_{ij}^{p}, Φ_{ij} and ε_{ij} for flows without body forces can be found in the literature (for a review, see e.g. Hanjalić, 1994).

The extra term S_{ij}^{M} in Eq. (5) represents the direct effects of fluctuating Lorentz force and involves the magnetic field $\mathbf{B}$ as well as second- and higher-moments of fluctuating velocity and fluctuating electric field $\mathbf{e}$ (or gradient of the fluctuating electric potential ϕ). The physical processes associate with these terms will be discussed below. However, additional indirect effects appear also in the pressure scrambling term Φ_{ij}, because the Poisson equation for pressure contains the Lorentz force.

The effects of body forces in the stress and dissipation equation can, in principle, be treated in the same way irrespective of the nature of the body forces. It is first noted that body forces do not affect directly the diffusion and viscous dissipation. Body forces act as a source or sink of turbulence and appear in the stress and dissipation transport equations as additional production (or redistribution) terms. They also appear in the pressure scrambling terms and these effect need to be accounted for in the model of Φ_{ij}. Implementation of the effects of common body forces in the second-moment closures, both at differential and algebraic level, have been proposed in the literature, e.g., for thermal buoyancy (Launder, 1975, 1989; Hanjalić, 1994), mass buoyancy (Hanjalić and Musemić, 1997) and rotation (Launder et al., 1987).

Because of preferential orientation, body forces usually affect only some of the components of the stress tensor, giving rise to stress anisotropy. However, the Lorentz force has a different character from other body forces. While system rotation and gravitational buoyancy may lead both to turbulence suppression and enhancement, Lorentz force is always dissipative. For example, system rotation causes only a stress redistribution and thus only an indirect effect on the kinetic energy (which may be either positive or negative), whereas the Lorentz force dissipates kinetic energy, though in a selective manner: it dampens only the stress components which are not aligned with the magnetic field vector. In both cases, however, the net effect leads to a two-dimensional eddy structure in a sense that the flow properties tend to be independent of one of the coordinates.

The strong and direct effect of body forces on stress anisotropy diminishes a priory the prospects for their proper accounting in the linear eddy-viscosity models: the problems of predicting rotating and buoyancy-driven flows with the standard k–ε models are well known. Nevertheless, most of commercial CFD codes use the k–ε model for various applications, with some ad hoc modifications for body forces. In view of the needs for simple models to compute complex metallurgical and other flows of conductive fluids in magnetic field, we propose here the modification of standard k–ε model, but based on the analysis of the parent full stress-transport equation. The analysis will also give an indication on the possible modifications within the framework of second-moment algebraic or differential models, though again our attention will be confined only to the treatment of the source term S_{ij}^{M}. Additional modifications for second-moment closure, involving the pressure–strain term Φ_{ij} will not be dealt with in the present paper. One of the reasons for omitting the discussion on the pressure–strain term is that the DNS of a plane channel flow by Noguchi and Kasagi (1994), used here to analyse and model these effects, do not provide separate information on slow, rapid and magnetic part of the pressure–strain term. We, therefore, confine our attention only to a deriving and validating models for the additional source terms S_{ij}^{M} and its simplification for the k–ε model.

2.1. The source term S_{ij}^{M} in $\overline{u_iu_j}$ equation

The implementation of the fluctuating Lorentz force into the stress transport equation leads to the following expression for the extra source term:

$$S_{ij}^{\mathrm{M}} = \frac{\sigma}{\rho}\left(\underbrace{-\epsilon_{ikl}B_l\overline{u_j\frac{\partial\phi}{\partial x_k}} - \epsilon_{jkl}B_l\overline{u_i\frac{\partial\phi}{\partial x_k}}}_{S_{ij}^{\mathrm{M1}}} + \underbrace{B_iB_k\overline{u_ju_k} + B_jB_k\overline{u_iu_k} - 2B_k^2\overline{u_iu_j}}_{S_{ij}^{\mathrm{M2}}}\right). \tag{6}$$

In order to illustrate the effect of the total $S_{ij}^{\mathrm{M}}$the term, we show in Table 1 both contributions, S_{ij}^{M1} and S_{ij}^{M2}, for each of the stress components in a fully developed channel flow with a transverse magnetic field $\mathbf{B} = B_y$.

The terms are nondimensionalised using the inner wall scales, U_τ and ν. The nondimensional form shows that both terms are multiplied by the interaction parameter $N = Ha^2/Re$, meaning that the terms are significant only if N is sufficiently large. Table 1 also shows that in this case the Lorentz force has no effect on the stress component aligned with the magnetic field.

The physical meaning of the second group of terms S_{ij}^{M2} can be further elucidated if we consider a general case with a constant magnetic field and rearrange the last term with the introduction of the stress component that is aligned with the magnetic field vector, $\overline{u_B^2}$ (Naot et al., 1990)

$$S_{ij}^{\mathrm{M2}} = \frac{\sigma}{\rho}\left(\underbrace{-\tfrac{2}{3}B_k^2(2k - \overline{u_B^2})\delta_{ij}}_{S_{ij,1}^{\mathrm{M2}}}\underbrace{-2B_k^2(\overline{u_iu_j} - \tfrac{2}{3}k\delta_{ij})}_{S_{ij,2}^{\mathrm{M2}}} + \underbrace{(B_jB_k\overline{u_iu_k} + B_iB_k\overline{u_ju_k} - \tfrac{2}{3}B_k^2\overline{u_B^2}\delta_{ij})}_{S_{ij,3}^{\mathrm{M2}}}\right). \tag{7}$$

Since $\overline{u_B^2}$ is unaffected by the Lorentz force, its is clear that the first term, $S_{ij,1}^{\mathrm{M2}}$, represents the decay of turbulence kinetic energy (in fact, of two stress components other than $\overline{u_B^2}$) forcing the turbulence to approach the one-component state and, eventually, to disappear. The next two terms have zero trace and are redistributive. The second term, $S_{ij,2}^{\mathrm{M2}}$, represents the stress isotropisation process by the magnetic field, and the third term, $S_{ij,3}^{\mathrm{M2}}$, is the stress-component redistribution with respect to the direction of the magnetic flux density vector.

It is noted that the complete S_{ij}^{M2} term can be treated in exact form (no modelling is required) in the second-moment closure approach where all stress components are computed. However, the term S_{ij}^{M1} needs to be modeled, except if a separate transport equation is solved for the correlation $\overline{u_ie_k} = \overline{u_i(\partial\phi/\partial x_k)}$.

Table 1
Specification of additional dimensionless magnetic terms in $\overline{u_iu_j}$ equations for fully developed plane channel flow under presence of transversal magnetic field ($S_{ij}^{\mathrm{M}} = S_{ij}^{\mathrm{M1}} + S_{ij}^{\mathrm{M2}}$)

	$\overline{uu}$	$\overline{vv}$	$\overline{ww}$	$\overline{uv}$
S_{ij}^{M1}	$2\frac{Ha^2}{Re}B_y^+\overline{u^+\frac{\partial\phi^+}{\partial z^+}}$	0	$-2\frac{Ha^2}{Re}B_y^+\overline{w^+\frac{\partial\phi^+}{\partial x^+}}$	$\frac{Ha^2}{Re}B_y^+\overline{v^+\frac{\partial\phi^+}{\partial z^+}}$
S_{ij}^{M2}	$-2\frac{Ha^2}{Re}B_y^+B_y^+\overline{u^+u^+}$	0	$-2\frac{Ha^2}{Re}B_y^+B_y^+\overline{w^+w^+}$	$-\frac{Ha^2}{Re}B_y^+B_y^+\overline{u^+v^+}$

3. The k–ε model for conductive fluid in magnetic field

The source-like terms appear also in the equation for turbulence kinetic energy $k = \overline{u_i u_i}/2$ and in the equation for energy dissipation rate ε, which is used to close the model equation set in both second-moment or k–ε eddy-viscosity models. Adopting the conventional approach, these two equations can be written as

$$\frac{\mathrm{D}k}{\mathrm{D}t} = P_k + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] - \varepsilon + S_k^{\mathrm{M}}, \tag{8}$$

$$\frac{\mathrm{D}\varepsilon}{\mathrm{D}t} = C_{1\varepsilon}\frac{\varepsilon}{k}P_k + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] - C_{2\varepsilon}\frac{\varepsilon^2}{k} + S_\varepsilon^{\mathrm{M}}. \tag{9}$$

The exact form of the additional source terms can be obtained from the conventional derivation (for the k equation this is simply half the trace of S_{ij}^{M})

$$S_k^{\mathrm{M}} = \frac{\sigma}{\rho}\left(\epsilon_{ijk}B_k\overline{u_i\frac{\partial\phi}{\partial x_j}} + B_iB_k\overline{u_iu_k} - 2kB_k^2\right), \tag{10}$$

$$\begin{aligned} S_\varepsilon^{\mathrm{M}} = &-\frac{2\nu\sigma}{\rho}\epsilon_{ijk}\left(\frac{\partial B_k}{\partial x_l}\overline{\frac{\partial u_i}{\partial x_l}\frac{\partial\phi}{\partial x_j}} + B_k\overline{\frac{\partial u_i}{\partial x_l}\frac{\partial^2\phi}{\partial x_l\partial x_j}}\right) \\ &+\frac{2\nu\sigma}{\rho}\left(B_k\frac{\partial B_i}{\partial x_l}\overline{u_k\frac{\partial u_i}{\partial x_l}} + B_iB_k\overline{\frac{\partial u_i}{\partial x_l}\frac{\partial u_k}{\partial x_l}} + B_i\frac{\partial B_k}{\partial x_l}\overline{u_k\frac{\partial u_i}{\partial x_l}}\right. \\ &\left. - B_k^2\overline{\left(\frac{\partial u_i}{\partial x_l}\right)^2} - 2B_k\frac{\partial B_k}{\partial x_l}\overline{u_i\frac{\partial u_i}{\partial x_l}}\right). \end{aligned} \tag{11}$$

In the k-equation the redistributive terms $S_{ij,2}^{\mathrm{M2}}$ and $S_{ij,3}^{\mathrm{M2}}$ disappear and the remaining term is purely dissipative, representing the decay of k. A physical interpretation of $S_\varepsilon^{\mathrm{M}}$ is much less transparent and will not be elaborated here. Some insight can be gained again from the inspection of terms for a special case of a plane channel flow with a uniform transverse magnetic field in normal (y) direction, as discussed earlier for the stress equation, Table 2.

We discuss now possible ways of deriving simple models for extra terms in both equations. Recently, Ji and Gardner (1997) proposed modifications to the standard low-Re number k–ε model to account for the magnetic field. They proposed additional terms in the k- and ε-equations, supposedly to model S_k^{M} and $S_\varepsilon^{\mathrm{M}}$, as well as a damping function f^{M} (in addition to f_μ) for turbulent viscosity, defined as $\nu_t = C_\mu f_\mu f^{\mathrm{M}} k^2/\varepsilon$. Both new terms and the additional function in ν_t contain exponential damping $\exp(-C_2^{\mathrm{M}}N)$ which they derived from an approximate analysis of the velocity decay under the action of a transverse magnetic field, $U(\tau) = U_0\exp(-t/t_m)$, by taking $t_m = \rho/\sigma B_0^2$ as the characteristic magnetic breaking time.

Table 3 shows the models for the terms S_k^{M} and $S_\varepsilon^{\mathrm{M}}$, as well as and for the additional damping function f^{M} as proposed by Ji and Gardner (1997) (denoted by JG). The JG model requires two new coefficients for which the following values were proposed: $C_1^{\mathrm{M}} = 0.05$ and $C_2^{\mathrm{M}} = 0.9$. It is noted that JG proposed to use the *bulk flow* Stuart number (the interaction parameter) $N = \sigma B_0^2 L/\rho U$ as a basic parameter to define the damping of turbulent quantities due to magnetic field. The use of bulk-flow parameters to model terms in equations for turbulence quantities is not a common practice in turbulence modelling as it makes the model dependent on bulk flow properties. The Stuart number defined above is constant for a particular flow for the imposed magnetic flux B_0 and, consequently, the JG proposal is limited to configurations for which it is possible to define an integral value of Stuart number, e.g., to relatively simple geometries (pipe or channel configuration) and homogeneous magnetic fields. Also, an ad hoc direct damping of turbulent viscosity due to applied external force has no physical justification, if k- and ε-equations are properly modelled.

In order to remove the above mentioned deficiency of the Ji and Gardner (1997) proposal, we replaced this integral value by a local interaction parameter which was obtained by introducing the local time scale ($\tau = k/\varepsilon$) in its definition (which had also been indicated by JG as a possibility). This new definition should reflect the local interaction of the characteristic magnetic breaking time and the characteristic turbulence time scale, so that the model should be able to deal with configurations of arbitrary geometries as well as with inhomogeneous magnetic fields. Additionally, the damping function f^{M} is omitted from our new proposal. These changes lead to the reduction of empirical coefficients to only one, C_1^{M}, which was evaluated through a priori term-by-term testing on the basis of the DNS database for the plane channel by Noguchi and Kasagi (1994). A summary of both the Ji and

Table 2
Specification of additional dimensionless magnetic terms in k- and ε-equations for fully developed plane channel flow under transverse magnetic field ($S_\phi^{\mathrm{M}} = S_\phi^{\mathrm{M1}} + S_\phi^{\mathrm{M2}}$)

	k	ε
S_ϕ^{M1}	$\frac{Ha^2}{Re}B_y^+\left(\overline{u^+\frac{\partial\phi^+}{\partial z^+}} - \overline{w^+\frac{\partial\phi^+}{\partial x^+}}\right)$	$2\frac{Ha^2}{Re}B_y^+\left(\overline{\frac{\partial u^+}{\partial x_l^+}\frac{\partial^2\phi^+}{\partial x_l^+\partial z^+}} - \overline{\frac{\partial w^+}{\partial x_l^+}\frac{\partial^2\phi^+}{\partial x_l^+\partial x^+}}\right)$
S_ϕ^{M2}	$-\frac{Ha^2}{Re}B_y^+B_y^+\left(\overline{u^+u^+} + \overline{w^+w^+}\right)$	$-2\frac{Ha^2}{Re}B_y^+B_y^+\left(\overline{\frac{\partial u^+}{\partial x_l^+}\frac{\partial u^+}{\partial x_l^+}} + \overline{\frac{\partial w^+}{\partial x_l^+}\frac{\partial w^+}{\partial x_l^+}}\right)$

Table 3
Specification of additional terms in the Ji and Gardner (1997) and NEW model

Model	S_k^{M}	S_ε^{M}	f^{M}
JG	$-\frac{\sigma}{\rho}B_0^2kC_1^{\mathrm{M}}\exp\left(-C_2^{M}N\right)$	$-\frac{\sigma}{\rho}B_0^2\varepsilon C_1^{\mathrm{M}}\exp\left(-C_2^{\mathrm{M}}N\right)$	$\exp\left(-C_2^{M}N\right)$
NEW	$-\frac{\sigma}{\rho}B_0^2k\exp\left(-C_1^{\mathrm{M}}\frac{\sigma}{\rho}B_0^2\frac{k}{\varepsilon}\right)$	$-\frac{\sigma}{\rho}B_0^2\varepsilon\exp\left(-C_1^{\mathrm{M}}\frac{\sigma}{\rho}B_0^2\frac{k}{\varepsilon}\right)$	1

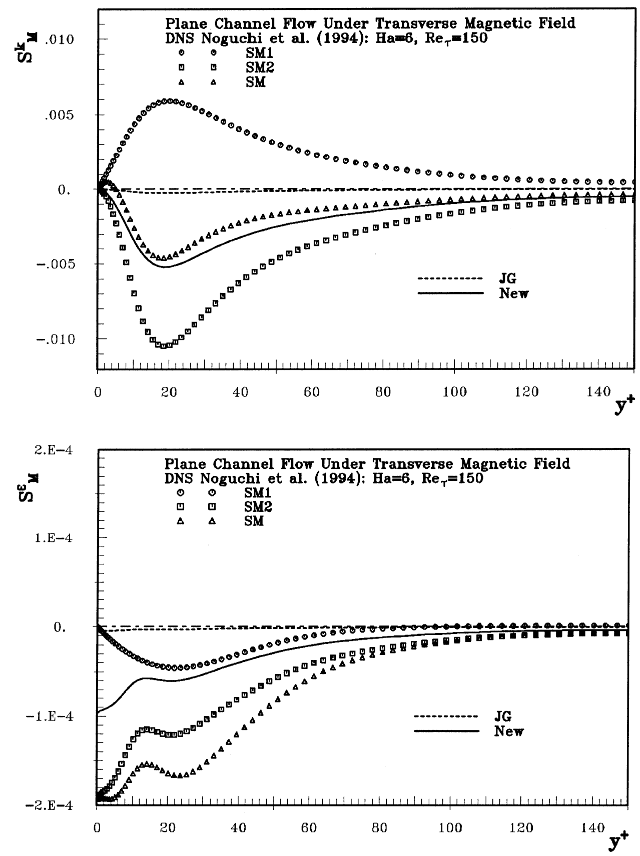

Fig. 1. Plane channel flow under transverse magnetic field, DNS by Noguchi and Kasagi (1994), $Re_\tau = 150, Ha = 6$, a priori testing of JG and NEW proposals for MHD-source terms in k- and ε-equations.

Gardner (1997) and of the here proposed new model of source terms in both the k- and ε-equations is given in Table 3.

In order to provide an argument for and a verification of the proposed modification, we consider the DNS results for the kinetic energy and dissipation budget in a plane channel flow, provided by Noguchi and Kasagi (1994). Fig. 1 shows magnetic source term contributions to the turbulence kinetic energy and dissipation rate budgets. In the DNS database this source contribution is decomposed in two parts, Table 2. The first part is associated with the terms that include velocity–electric field correlation, $\overline{u_i e_j} = -\overline{u_i \partial\phi/\partial x_j}$, ($\circ$), and the second, which include the remaining terms ($\Box$).

The lines in Fig. 1 show the validation of the proposed model, denoted by NEW, and the JG model. As seen, the JG proposal heavily underestimates the total source terms in both the k-equation and ε-equation. The NEW proposal with $C_1^M = 0.025$ agrees very well with the total DNS contribution ($\triangle$) for turbulence kinetic energy. For dissipation, agreement is qualitatively good, but the DNS value is underestimated in the near wall region. The JG proposal also underestimates the value of dissipation rate obtained by DNS across the entire channel cross-section.

4. The second-moment closure with Lorentz force

Both of the above mentioned proposals were then extended to the RSM model, by replacing k with $\overline{u_i u_j}$ in Table 3. For a plane channel flow under transverse magnetic fields, only the $\overline{uu}, \overline{ww}$ and $\overline{uv}$ stress components include directly magnetic source terms, Table 1.

The model based on JG proposal again largely underpredicts the total magnetic source contribution for all stress components, Fig. 2. In contrast, the NEW model shows a qualitatively good behaviour for the normal stress components, but the shear stress is overestimated. The model for S_{uv}^M actually reproduces the dissipative contribution, S_{uv}^{M2}, Eq. (6).

In order to improve the model of magnetic source contribution to the shear stress, we decided to model both its contributions separately, i.e., S_{ij}^{M1} and S_{ij}^{M2}. For the plane channel flow, the S_{ij}^{M2} contribution can be treated exactly (the expression reduces to the product of the imposed magnetic field and the particular stress component, $-2Ha^2/Re\ \overline{u_i u_j}\ B_0^2$).

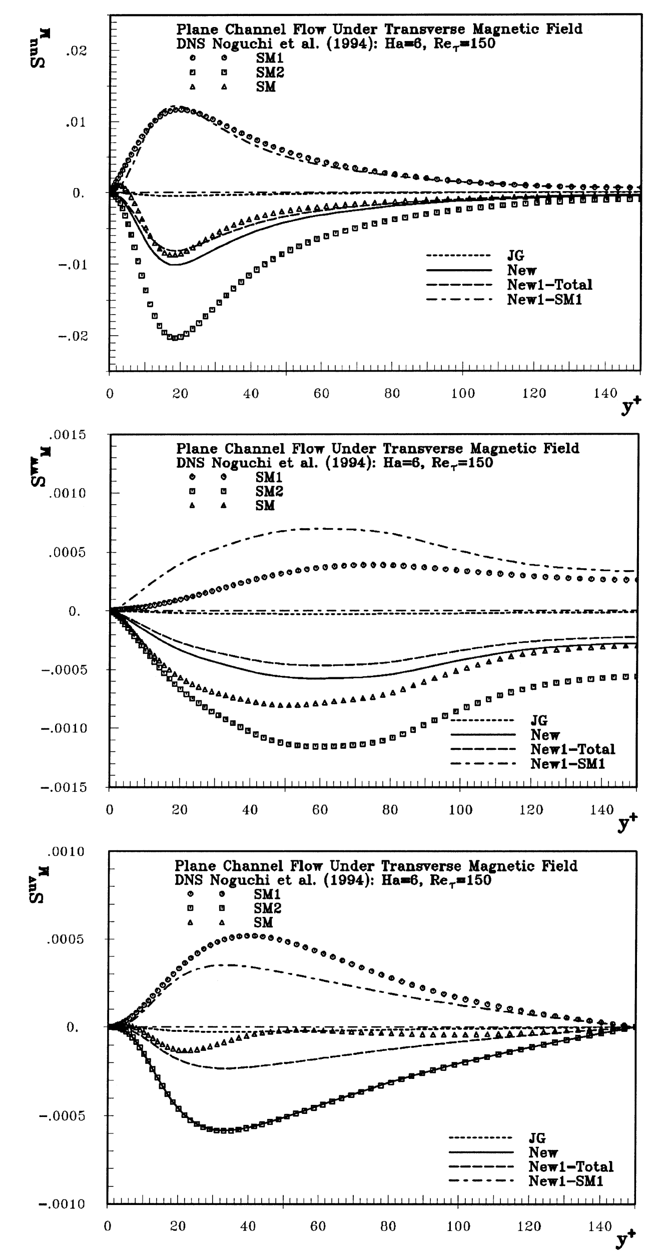

Fig. 2. Plane channel flow under transverse magnetic field, DNS by Noguchi and Kasagi (1994), $Re_\tau = 150, Ha = 6$, a priori testing of JG, NEW and NEW1 proposals for the MHD-source term (S_{ij}^M) in expanded RSM.

Consequently, it remains only to model S_{ij}^{M1}. As seen from Eq. (6), the correlation of velocity and electric field fluctuations ($\overline{u_i e_j} = -\overline{u_i \partial\phi/\partial x_j}$) determines the S_{ij}^{M1} contribution. After deriving the parent equation for $\overline{u_i e_j}$ correlation (by substituting Ohm's law for moving media in the equation for the magnetic field), and reducing the model to a plane channel flow, the following form of the model for S_{ij}^{M1} emerges:

$$\overline{u_i \frac{\partial \phi}{\partial x_j}} = C_\phi \epsilon_{jkl} B_l \overline{u_i u_k}, \tag{12}$$

where $0 < C_\phi \leqslant 1$ remains to be determined. The value of $C_\phi = 0.6$ was determined by optimising the S_{uu}^{M1} contribution, since this Reynolds stress component is the largest, Fig. 2(a). As seen, the new splitting procedure improves the prediction of magnetic source term contribution for the shear stress significantly, and retains good agreement for normal stress components.

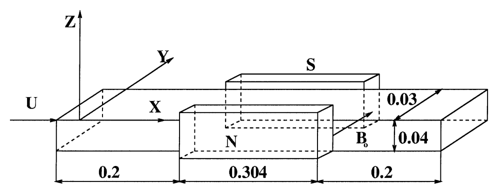

Fig. 3. Schematic representation of the flow (U) entering and leaving magnetic field (B_0) causing complex magneto-hydrodynamical interactions: $\mathbf{U} \times \mathbf{B} \rightarrow j_{\text{ind}} \rightarrow \Phi \rightarrow j_{\text{cond}} \propto E \rightarrow \mathbf{J} \times \mathbf{B} \rightarrow \mathbf{F}_\text{L}$.

5. Duct flow of liquid metal in an inhomogeneous magnetic field

The majority of liquid metal flows of practical relevance occur in the presence of inhomogeneous magnetic fields. This inhomogeneity originates mainly due to a limited size of the magnet. The effect of inhomogeneity of the magnetic field on the flow of a conducting fluid was examined in the configuration shown in Fig. 3. Liquid metal (mercury) enters a perpendicular magnetic field of strength (B_0) with uniform velocity (U). As a result, the current is induced in the liquid metal. This induced current (j_{ind}) leads to an electric potential (Φ) which generates a current due to the electric conductivity of the liquid metal (j_{cond}). This conductive current counteracts the induced current, resulting in a relatively small difference between them in terms of the total current (**J**). The total current combined again with the magnetic field ($\mathbf{B} = B_0$) results in a strong Lorentz force.

In order to investigate the validity of the numerical implementation of magnetic effects in the computational model, we considered the above mentioned flow of liquid metal through a rectangular duct with electrically insulating walls entering and leaving a uniform magnetic field. The magnetic field induces characteristic M-shaped velocity profile in the plane normal to the imposed magnetic field. This test case is based on the experimental investigation of Tananaev (in Branover, 1978), and is defined by the following characteristic nondimensional parameters: $Re = UL/\nu = 2 \times 10^5$, $Ha = \sqrt{NRe} = 700$, $N = \sigma B_0^2 L/\rho U = 2.45$.

Numerical computations were performed by a finite-volume Navier–Stokes–Maxwell solver for three-dimensional flows in structured, nonorthogonal geometries, with Cartesian vector and tensor components and colocated variable ar-

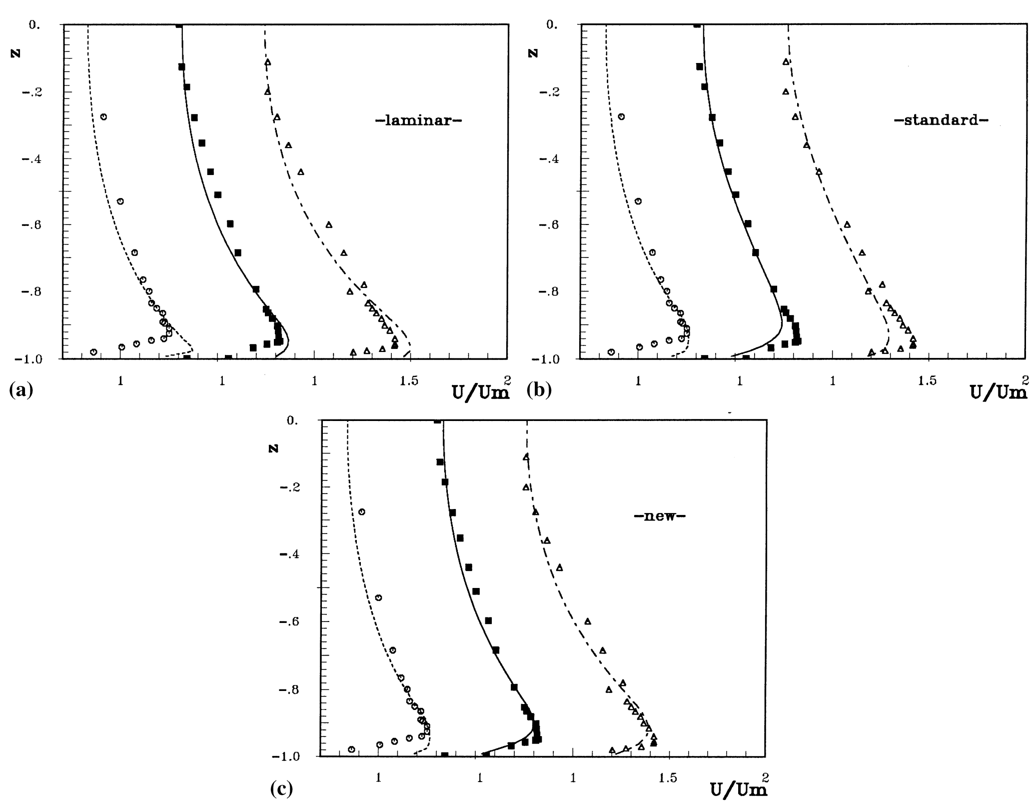

Fig. 4. Velocity profiles in a 3D MHD flow in a square duct, $Re = 2 \times 10^5, Ha = 700, N = 2.45$, experiment by Tananaev (in Branover, 1978), (−−, ○) magnet inlet ($x = 0.2$ m), (−, □) magnet half ($x = 0.352$ m), (−·, △) magnet outlet ($x = 0.504$ m) , (a) laminar, (b) standard k–ε model with F_L in momentum equation, (c) new k–ε model.

rangement and with wall functions. The grid resolution was $102 \times 43 \times 62$ grid nodes in x-, y- and z-direction, respectively. The fine grid distribution was applied in the direction perpendicular to the imposed magnetic field in order to resolve the Hartmann boundary layers which develop along the duct walls. The diffusion terms were treated by the central differencing scheme and the convective terms by using the QUICK scheme. At the inlet of the channel, the velocity was assumed to be uniform with a small uniform turbulence level (Fig. 6).

Fig. 4 shows a streamwise velocity profiles at three different locations. Despite a very high value of *Re* number, very strong magnetic field suppresses the turbulence and almost laminar profiles are obtained in the central part of the magnet. The velocity profiles obtained from the computations in the laminar regime with F_i^L in momentum equation, show a very similar behaviour with experiment. The formation of the characteristic M-shape of the velocity is captured in accordance with the experiment, but results show a constant overestimation of the velocity profiles indicating that turbulence is not completely damped. This is probably due to a relatively short length of the magnet as well as the high value of the *Re* number of the incoming flow. The calculations with the standard k–ε model with F_i^L in momentum equation produced velocity profiles which show lower peak values (due to a better mixing) than the corresponding experimental profiles, indicating that the produced level of turbulence kinetic energy is too high. Only the application of the NEW model which includes also the additional magnetic source terms in the k- and ε-equations, brings the predicted values in close agreement with experiment (except at the entrance of the magnet). This

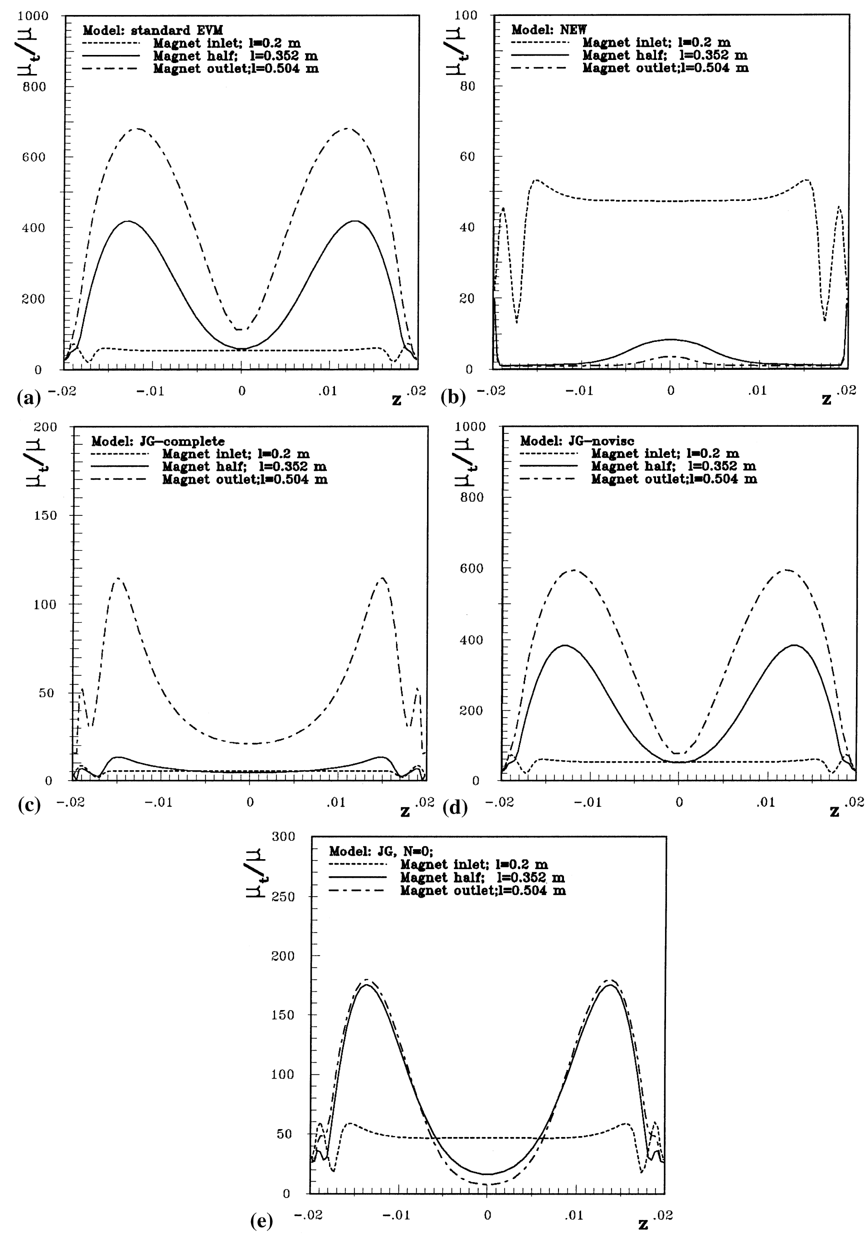

Fig. 5. Turbulent viscosity profiles in a 3D MHD flow in a square duct, $Re = 2 \times 10^5$, $Ha = 700, N = 2.45$: (– –) magnet inlet ($x = 0.2$ m), (–) magnet half ($x = 0.352$ m), (–·) magnet outlet ($x = 0.504$ m), (a) standard k–ε model with F_L in momentum equation; (b) new k–ε model; (c) JG model complete; (d) JG model without turbulent viscosity damping; (e) JG model with $N = 0$.

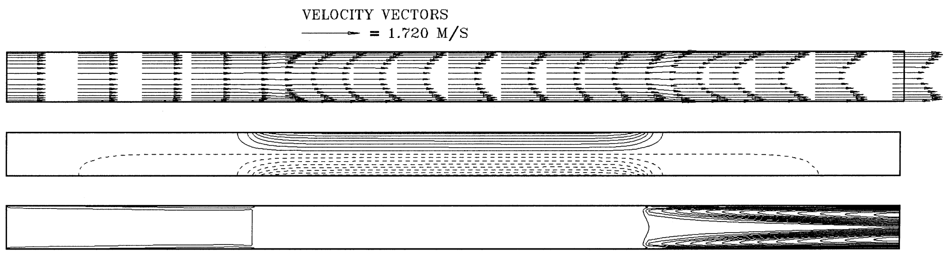

Fig. 6. The flow of liquid metal through a uniform magnetic field in rectangular duct with isolating wall; $Re = 2 \times 10^5, Ha = 700, N = 2.45$; the velocity vectors, electric potential and turbulent viscosity distributions.

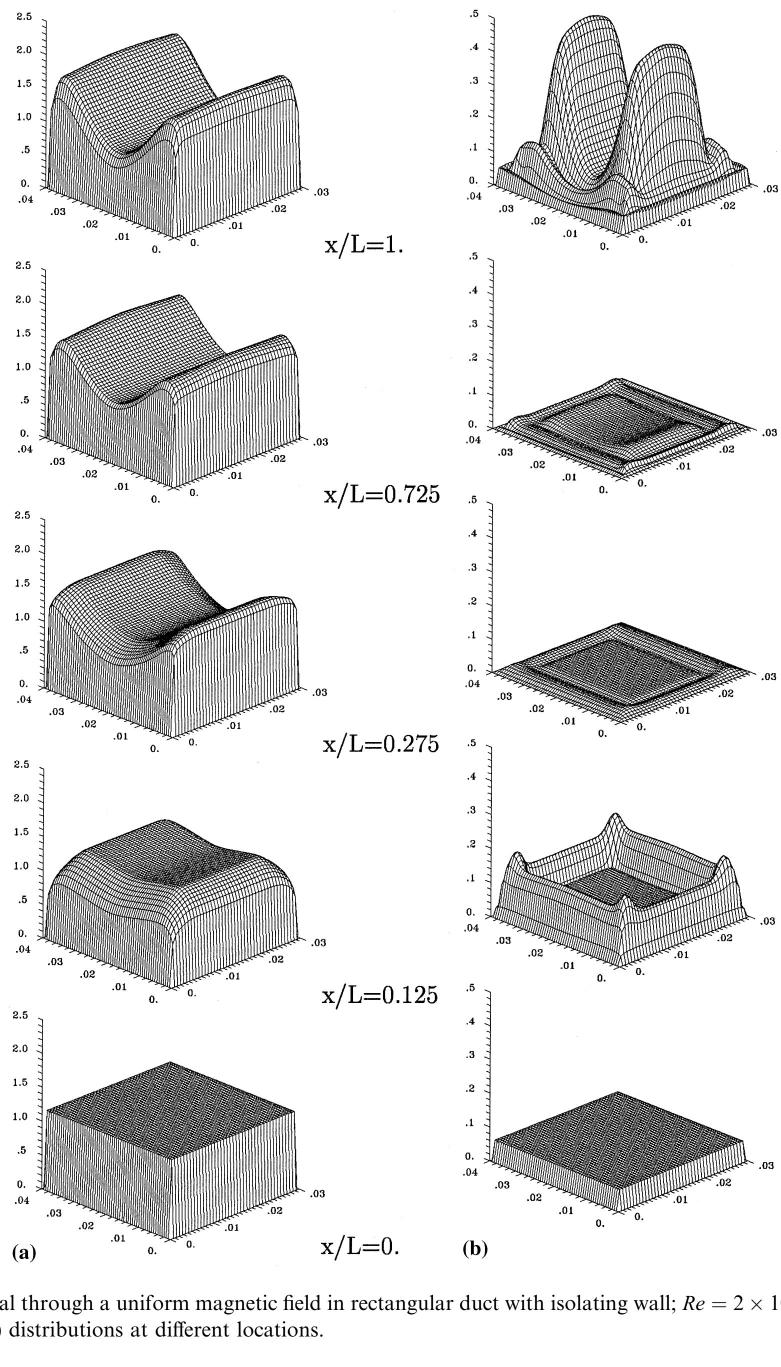

Fig. 7. The flow of liquid metal through a uniform magnetic field in rectangular duct with isolating wall; $Re = 2 \times 10^5, Ha = 700, N = 2.45$: velocity (a) and turbulent viscosity (b) distributions at different locations.

means that the ability of the model to predict a low level of turbulence is essential for this type of the flow. The calculations were performed also with the JG model, and surprisingly, similar agreement with experimental values was obtained as with the NEW model.

In order to clarify this behaviour, we deactivated first the additional damping function in the turbulent viscosity, by specifying $f^M = 1$, and the final results were very similar to the standard version of k–ε model. By deactivating the additional magnetic source terms and retaining the damping of turbulence viscosity, the effects on the final solutions were negligible in comparison with the full JG model. This leads to the conclusion that JG model affects turbulence mainly through the additional damping function in the turbulent viscosity, Fig. 5. This is in full accordance with our earlier findings which resulted from a priori testing where the magnetic source contributions were heavily underestimated in the JG model.

Fig. 6 shows the development and formation of the characteristic M-shaped velocity profiles as well as velocity vectors, electric potential and turbulent viscosity contours in the central plane $(x, y = 0, z)$ of the rectangular duct, shown in Fig. 3. Three-dimensional distributions of the mean velocity and eddy viscosity over the channel cross-section at several characteristic location prior, within and after the magnet, are shown in Fig. 7. It is interesting to note that the velocity profiles become slightly deformed even before the fluid reaches the channel part exposed to the magnetic field, due to the back effects of both the pressure and of electric field. In addition to the strong deformation of the mean velocity field at the magnet entrance, a second interesting feature is visible at the magnet exit, where the velocity profile is additionally deformed by the strong gradient of the magnetic field in the horizontal direction. The electric potential exhibits a linear distribution in z-direction with a negative value in the lower part, and positive ones in the upper part of the channel. The turbulent viscosity profiles indicate that the turbulence is strongly damped in the magnet region, but it does not vanish totally. In the region downstream from the magnet, the turbulence quickly recovers due to strong shear.

6. Conclusions

It has been demonstrated that the effect of a magnetic field can be accounted for in turbulence closure models by introducing source terms due to Lorentz force in the same manner as is practiced to model other body forces, e.g., thermal buoyancy. The new terms in the second-moment closure showed the same behaviour as those obtained by DNS for a flow of conducting fluid in a plane channel. This a priori validation enabled also the determination of a new empirical coefficient. The standard low-Re number k–ε model with the inclusion of the same model terms to account for magnetic field reproduces well the development of the M-shaped velocity profile in the high-Re number developing flow of liquid metal in a rectangular-sectioned duct subjected to a finite-length uniform magnetic field.

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